Geometry of the Fermat's spindle-shaped channel of an individual wave between its emitter and absorber

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Abstract

The space between the emitter and the absorber of an individual wave in the vacuum (photon, electron, neutron, neutrino, etc.) is symmetrized by the hypothesis that the curvature of the wave fronts is the difference between the inverses of the distances to the emitter and the absorber. Then, on a sagittal plane, the so-defined circles are precisely a Poncelet's bundle, orthogonal to the secant circles. The Fermat-Fresnel constraint on each actual route is that it does not delay more than a quarter of a period. It yields the maximum diameter of each Fermat's spindle-shaped channel: $\sqrt{3\lambda.a}\ /\ 2$, where $2\boldsymbol{a}$ is the distance between the absorber and the emitter, and λ is the wavelength. Let $\boldsymbol{r}$ be the radius of the physical apex, emitter, or absorber. It is seen from the mathematical apex at a Fermat's angle $\alpha = \sqrt{\frac{3\lambda}{4a}}$, so at a distance $\boldsymbol{\epsilon} = \boldsymbol{r}\ .\ \sqrt{\frac{4a}{3\lambda}}$. On astronomical distances, the individual character of the bosonic wave is no longer valid, but is dominated by the collective interaction of the bosons sharing similar frequency and similar wave vector. So is the Hanbury Brown & Twiss grouping, which permits interferential Astronomy.

Géométrie du canal-fuseau de Fermat d'une onde individuelle entre émetteur et absorbeur

Résumé

L'espace entre émetteur et absorbeur d'une onde individuelle (photon, électron, neutron, neutrino, etc.) est symétrisé en posant que la courbure des fronts d'onde dans le vide est la différence des inverses des distances respectivement à l'émetteur et l'absorbeur. Or les cercles ainsi définis par la trace des fronts d'onde sur un plan sagittal sont exactement un faisceau de cercles de Poncelet, orthogonaux aux cercles sécants. Ne pas prendre plus d'un quart de période de retard, limite la géométrie et le diamètre maxi de chaque canal-fuseau de Fermat à $\sqrt{3\lambda.a}\ /\ 2$, où $2\boldsymbol{a}$ est la distance entre émetteur et récepteur, et λ la longueur d'onde. Un apex physique émetteur ou absorbeur de rayon $\boldsymbol{r}$ est vu depuis l'apex mathématique sous un angle de Fermat $\alpha = \sqrt{\frac{3\lambda}{4a}}$, donc à distance $\boldsymbol{\epsilon} = \boldsymbol{r}\ .\ \sqrt{\frac{4a}{3\lambda}}$. Sur des distances astronomiques, l'individualité postulée est dépassée par le caractère collectif des bosons de fréquence et de vecteur d'onde proches. C'est le groupage Hanbury Brown & Twiss, utilisé en astronomie interférentielle.

1 Syntax of individual waves

An unsolved problem of Transactional Sub-quantum Microphysics was the exact geometry of Fermat's spindle-shaped channels, for any individual wave – such as photon, electron, neutron, neutrino, etc. – between the emission reaction and the absorption reaction. There was a provisional approximation, but it was not yet validated.

In Transactional Sub-quantum Microphysics, we state the principle that every photon has an absorber, hence the definition of the photon: a successful transaction between three partners, an emitter, an absorber and the optical space that separates them, which transfers by electromagnetic means a quantum of looping of Planck (6.6260755. 10^{-34} joule/hertz), and respectively an energy-momentum which depends on the respective frames of the emitter and the absorber. Historically, this notion arose from spectrography, when at least one of the emitter or the absorber, oscillating from an initial stationary state to a stationary final state, is therefore bound by the resonance rules dependent on the quantum of Planck **h**, governed by Schrödinger's matter wave equation [1] and its successor Dirac's equation of the electronic wave [2].

Only the Retrosymmetry between emitter and absorber can obtain the directivity of each photon, proven by

Albert Einstein [3] in 1916: the emitter alone is very, very small in front of the wavelength, therefore incapable of the slightest directivity, while the emitter-absorber coupling and the successful handshake between them, at a distance, obtain this directivity.

2 Fermat's principle, with wavelength

In the time of Pierre de Fermat, the wavelengths of light were unknown, and so were interferences. If Fermat's principle is reformulated with periodicity and wavelength, it gives "Everything that is emitted in phase arrives in phase at the absorber, by very close paths." Since interference has been practiced, for instance in radiocrystallography, we add "in phase, to within an integer of periods." But let us stay here on the case of the single path in the sense of geometric optics, when the radiocrystallography was unknown, and the interference colors were inexplicable. Fermat's principle implies a finite and non-zero width of the channel occupied by the photon (or any other quanton), also the maximum envelope of this path is that which would give a delay of a quarter of a period, beyond which the interference becomes destructive, and the transmitted power is zero.
Does this mean that nothing matters anymore beyond the thin Fermat's spindle-shaped channel? The monopolistic tradition professes something entirely different. Well, since 1927, this monopolistic tradition confuses and amalgamates the path accomplished when a transaction succeeds – a steep path, constrained by steep phase fronts – with all the background noise of the broglian waves [4], which have all palpated (transmittances inclusive) through countless orthochronous and retrochronous micro-times. Each successful transaction emerges from the Dirac-de Broglie background noise, and these successful emergences are comparatively rare, compared to all the partial resonances that abort (and that we will not detail experimentally: the Ashby's Theorem of the Requisite Variety [5] restraints our panoptical dreams of unlimited knowledge).
The countless articles and debates debating "*Welchen Weg?*" (Which way?) confirm that the surreptitious corpuscularism remains inherent in the monopolistic tradition. Now, it is not only invalidated by all the interferences known since the 19th century, invalidated by the existence of plane-polarized light over great distances, invalidated by the laws of birefringence in anisotropic crystals, invalidated by all modes of radiocrystallography (including electronic and neutronic ones [Rauch+Treimer+Bonse74][Treimer2024]), invalidated by the behavior of interferential colors and anti-reflection layers at large angles. Moreover, this implicit corpuscularism is invalidated by the interferential transparency of gas to the slow electrons, discovered independently in 1921 by Carl Ramsauer [6] and by S. Townsend and V. A. Bailey [7], amply confirmed since then. The best known of the definitive invalidations of surreptitious corpuscularism was experimentally proven by Shahriar S. Afshar [8], since 2004. Never does any individual wave transmute into some corpuscular thing. The corpuscularism may be the most toxic among the twenty-eight surreptitious postulates that burden the monopolistic tradition for 1927. Many essential knowledges never cross the distance between two lecture rooms on the same campus.

3 Retrosymmetry emitter | absorber

Throughout the teaching of electromagnetism, waves are represented with a small-sized, even punctual emitter, and diluting throughout space towards an infinity of absorbers. Meanwhile, in Transactional Microphysics, any description omitting the absorber is doomed as *ipso facto* invalid. Therefore, we must transform the space so that it reintegrates the absorber into the same causal rank as the emitter. In the standard representation, the radius of curvature of the wavefronts is equal to the distance to the emitter. In other words, the curvature is the inverse of the distance to the transmitter.
In Retrosymmetry, the minimalist physical hypothesis is that in a vacuum, the curvature of the wavefront is the difference between the two inverses of distances to the emitter and absorber. So there is a null curvature at mid-distance, and infinite curvature at emitting or absorbing apex.
Transforming the curvature into the radius: $\mathbf{R_{y=0} = (r_a \; . \; r_e) \; / \; (r_a \text{ - } r_e)} = \frac{r_a . r_e}{r_a . - r_e}$
As long as we take the first half of the beam, on the emitter side, and very close to the optical axis, this radius is positive: from the center to the wavefront of the outgoing beam.
All the real arcs of enlargement of the photon are small arcs of this circle. The arrow is half the wide of the spindle-shaped channel. Let 2**a** be the distance between the two apexes.
The abscissa is taken from the middle of the optical axis of the spindle-shaped channel. We denote **e** the abscissa on the axis.
The real, physical spindle-shaped channel then extends from $\mathbf{-a+\varepsilon}$ to $\mathbf{+a-\varepsilon}$,
where $\boldsymbol{\varepsilon}$ is the shift of the physical apex (atom, for example, dye molecule, etc.) to the mathematical apex, slightly further away.
Wavefront radius: $\mathbf{R(e) = (a+e).(a\text{-}e)/\text{-}2e = (a^2\text{-}\ e^2)\ /\text{-}2e = (e^2\text{-}\ a^2)\ /2e = \frac{1}{2}\ (e\ \text{-}\ a^2/e)}$
Abscissa of the center of the circle: $\mathbf{x_c = e\ \text{-}\ R(e) = e\ \text{-}\ (e^2\text{-}\ a^2)/2e = (2e^2 + a^2\ \text{-}\ e^2)/2e = (a^2+\ e^2)}$

$/2e = \frac{1}{2}\ (a^2/e + e)$
Limit when $\mathbf{e} = \mathbf{-a}$: $\mathbf{x_c} = \mathbf{-a}$.
Equation of the circle:
$\mathbf{y^2 = 2(x\text{-}e + (e^2\text{-}\ a^2)\ /2e).\ (a^2\text{-}\ e^2)\ /2e\ \text{-}\ (x\text{-}e)^2}$
Abscissa of the other intersection of the circle with the extended optical axis: $\mathbf{a^2/e}$
Now, let's compare these two divisions of the emitter-absorber segment:
$-\ (\boldsymbol{a} + \boldsymbol{e})\ /\ (\boldsymbol{a}\ \text{-}\ \boldsymbol{e})$ and $((\mathbf{a^2}/\boldsymbol{e}) + \boldsymbol{a})\ /\ ((\mathbf{a^2}/\boldsymbol{e}) + \boldsymbol{a}) = (\boldsymbol{a} + \boldsymbol{e})\ /\ (\boldsymbol{a}\ \text{-}\ \boldsymbol{e})$
This was the divine surprise that dispenses us with all the gradient calculations and its integration: all these circles perform a harmonic division of the segment joining the two apexes, so in other words, the circles representing the wavefronts on a sagittal plane are a Poncelet's bundle of circles, orthogonal to the secant circles. We have met again the bundle of orthogonal circles of Apollonios from Perga (3rd – 2nd century BC).

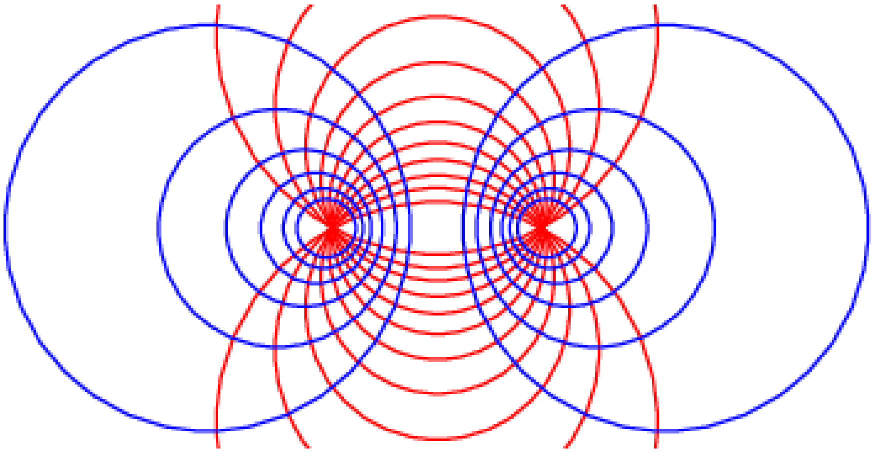

Figure 1. Bundles of orthogonal circles of Apollonios

Therefore, real journeys in a vacuum are arcs of circles: constant curvature from the physical emitting apex to the physical absorber apex. But this constant curvature is very small. Let's check the orders of magnitude, and the valid approximations.

4 Relative order of magnitudes of the wavelengths and diameters of the apexes?

Case of the Mössbauer's radiation of ^{57}Fe: $\lambda = 86.1$ pm $= 86\ 100$ fm. The known diameter of this nucleus is of the order of 10 fm. This yields a ratio of about 1 to 9,000 of the emitting or absorbing apex diameter to the photon wavelength. Given the ultra-fine frequency definition of this photon, this involves some ten billion to one hundred billion nucleus oscillations between the final state and the initial state to emit an entire photon, or to receive it in its entirety. This also involves several meters, or even about ten meters in length of coherence of the photon.
The calculation can be repeated for the selective carbon monoxide absorption line at 65.05 Terahertz: 4.608 µm / 0.47 nm $\approx$ 10,000, at the accuracy near this diameter of the CO molecule. We fall on the same order of magnitude of the ratio [wavelength/apex diameter].
The next question is to compare this wavelength to the total optical path. See the severe case of the carbon monoxide detector, with about 23 cm of total optical path of 50,000 wavelengths. However, in a Fermat's spindle-shaped channel between emitter and absorber, no more than a quarter wavelength of additional journey is allowed, that is one over two hundred thousand. In visible light, it would be about 500,000 wavelengths for the same optical path.

5 Series Expansion

It will be sufficient to rely on the second order of limited development, which approximates the arc of a circle by a parabola, slightly increasing the diameter in the middle of the channel.

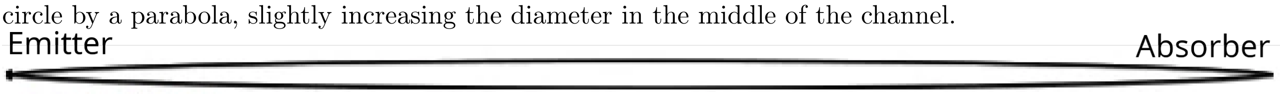

Figure 2. Fermat's spindle-shaped channel, simple case

Where $2\boldsymbol{z}$ is the diameter in the middle of the spindle, $2\boldsymbol{a}$ is the distance between the emitter and the receiver, considered as punctual.
The condition of Fermat's spindle-shaped channel is written: $\mathbf{2}\ \alpha\ \boldsymbol{R} - \mathbf{2}\ \boldsymbol{R}\ \mathbf{sin}(\ \alpha) < \frac{\lambda}{4}$.
Or in the first order: $\alpha^{\mathbf{3}} < \frac{\lambda}{4R}$, or $\boldsymbol{a} = \boldsymbol{R}.\mathbf{sin}(\alpha)$, which in the first term for very small angles is equivalent to $\boldsymbol{R}.\alpha$.
It is then possible to eliminate the radius $\boldsymbol{R}$ of the circle, and in the first non-zero term of the series expansion it remains: $\boldsymbol{z}^{\mathbf{2}} = \mathbf{3/16}\ \boldsymbol{a}.\lambda$ where $\boldsymbol{\lambda}$ is the wavelength.
We take the square root: $\boldsymbol{z} = \sqrt{3\lambda.a}\ /\ \mathbf{4}$.
Express it in relation to wavelength: $\frac{z}{\lambda} = \sqrt{\frac{3a}{16\lambda}}$
Let's assess the error due to limited expansion. A more analytical calculation had been published in 2000 and 2003:
$\left(\frac{z}{a}\right)^2 = \frac{3q}{16}(\mathbf{1} + \frac{3q}{80} + \frac{1233q^2}{448000} + \ldots)$.
Where $\boldsymbol{q}$ is the quotient of the wavelength by $\boldsymbol{a}$.
However, in infrared laboratory optics, this quotient $\boldsymbol{q}$ is of the order of 1/50 000. hence a first-order error of 0.75 ppm on the square of $\boldsymbol{z/a}$, and above 0.38 ppm of error in $\boldsymbol{z/a}$. Negligible.
There is also a need for the half-angle at the top of the tangent cone, close to emission or absorption reactions, or Fermat's angle: $\alpha = \sqrt{\frac{3\lambda}{4a}}$

6 The Paradox over Astronomical Distances

What about, with the distance from the Earth to the Moon? Distance $2\mathbf{a} = 384{,}000$ km. We take in the visible frequencies: $\boldsymbol{\lambda} = 0{,}5$ µm.
$3\ .\ \boldsymbol{a}\ .\ \lambda = 576\ \mathrm{m}^{\mathbf{2}}$
$\sqrt{3\lambda.a} = 24$ m.
To be divided by 2 to have the maximum diameter of the Fermat's spindle-shaped channel: 12 m. That is of the order of ten or twelve times the coherence length. This distance is about 150 times too large for us to be still able to speak of **individual** waves for each photon – here in the visible domain.
So, let's set the upper validity limit (of the individual photonic wave channel geometry in the visible range) around 2,000 km, corresponding to low Earth orbits. By way of comparison, the radius of the geostationary orbit is 42,164 km, or 35,786 km above the ground of the equator.

7 Very large base interference astronomy, and Hanbury Brown and Twiss bosonic grouping

When we calculate Fermat's spindle-shaped channel diameters over astronomical distances, we find diameters that also become almost astronomical. As a result, the reason why wide-base interference astronomy is possible is that these photons, which arrive in detectors tens of miles away on Earth, had plenty of time to synchronize during their shared paths, where they broadly shared their propagation widths, which may exceed the diameter of a star, or even an astronomical unit. The space traveled by is of disproportionate importance compared to the emitting and absorbing apexes.

Sun is about 149.6 million km away. We neglect the Earth's radius; we subtract three-quarters of the solar radius, resulting $1.492\ .\ 10^{11}$ m. The angle at the Fermat cone is about 0.8 µrad. The maximum width of the Fermat's spindle-shaped channel from a small emitter reaches 192 m. On the one hand, the bosonic character of the sun photons [9] is amply justified. On the other hand, this bosonic character over astronomical distances is powerful enough to invalidate the individual widths calculated for Fermat's spindle-shaped channels in interaction, not because these widths become false, but because the implicit hypothesis of individuality is invalidated: among bosons, the space is shared, collectivized.

8 Offsetting the mathematical apex beyond the physical apex

We have no correct theory of the nearby field, around the emitting atom or the absorbing atom, to remain in the historical cases of spectrography. We are only sure that the totality of the volume of the final state, and of the initial state, and especially their periphery, where they differ the most, intervene.
Solution: Remote the theoretical mathematical apex beyond the physical apex (atom or molecule), so that the tangent cone has precisely the diameter (blur) of the physical apex. Well, how much further?
Let us return to the case of the Earth-Moon journey, and calculate the Fermat angle of the tangent cone in

vacuum, with $\boldsymbol{\lambda} = 0.5$ microns and $\boldsymbol{a} = 192{,}000$ km.
Fermat angle: $\boldsymbol{\alpha} = \sqrt{\frac{3\lambda}{4a}}$, here $\alpha = \sqrt{\frac{1{,}5.10^{-6}}{768.10^{6}}} = 44\ .\ 10^{-9}$ radians.
To see a $\boldsymbol{r} = 200$ pm atomic radius in this angle, it is necessary to set back from $\boldsymbol{\epsilon} = \boldsymbol{r}/\alpha = 4{,}5$ mm. It is imperceptible over such a distance, much smaller than the uncertainties on the distance $2\boldsymbol{a}$ measurements, and much smaller than the non-definitions of altitude by irregularity of the lunar soil.

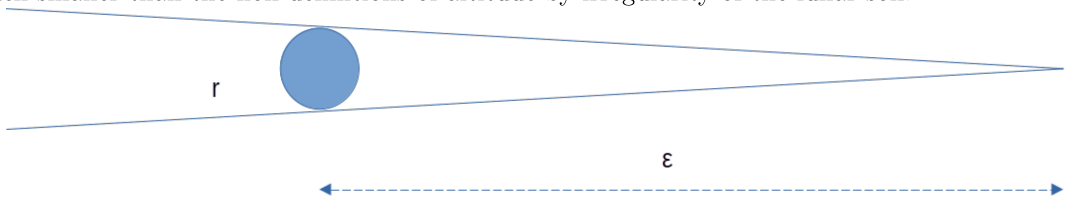

Figure 3. Distance between physical apex and mathematical apex

General formula of the mathematical apex offset off physical apex:
offset $\boldsymbol{\epsilon} = \boldsymbol{r}\ .\ \sqrt{\frac{4a}{3\lambda}}$
Therefore, this offset varies as the square root of the emitter-to-absorber distance, with an equal wavelength. In all cases, it is minimal, and hardly noticeable in any digital application. Consistency is verified with the initial hypotheses: the radius of the wavefront at the physical apex, here an atom of 4 Å in diameter, oscillating between a final state and an initial state, is practically $\boldsymbol{\epsilon} = 4.5$ mm, or 22 million times greater than the radius of this physical apex. And about 900 times the wavelength. So, we remain temporarily exempt from a near-field theory.*

9 Transverse density of presence in the section of a Fermat's channel

A glance at the figure of the Apollonios' circles confirms the shapes of the quasi-wavefronts dividing the journey, so the contribution to dephasing the out-of-the-axis part of propagation: They are spheres, approximated by paraboloids. So
dephasing/maxi_dephasing = $(z/z_{max})^2$
Continuing the simplifying model of propagation, we hypothesize that the contribution to the field projects like a cosine of the dephasing.

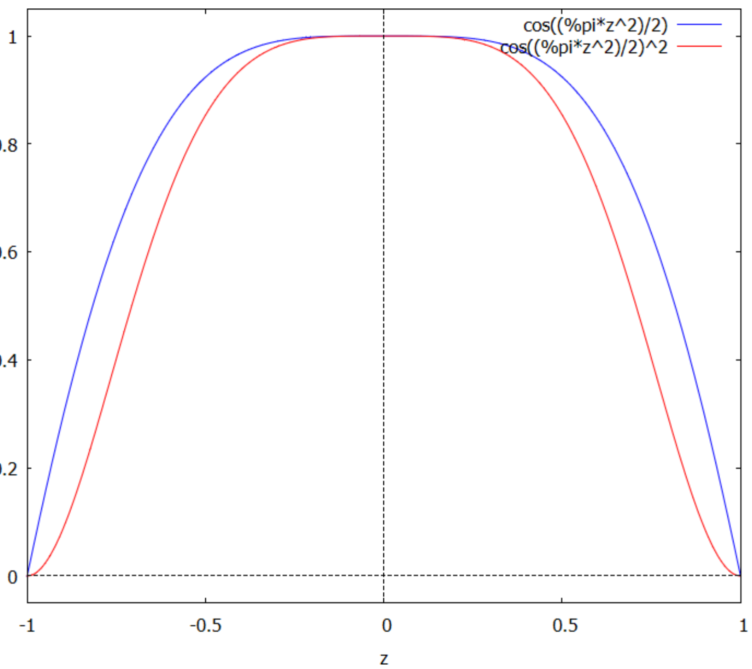

Figure 4. Radial occupation of a Fermat's channel. Simplified.

On the same graph, we have also represented the modulus of the Poynting vector, the contracted product $\frac{1}{\mu_0}\breve{B}.\vec{E}$ ($\vec{E}$ is a vector, $\breve{B}$ is a gyror, or antisymmetric tensor), in comparison with the maximum modulus in the central axis of the channel.
Finding a more precise lateral distribution of the field and, subsequently, of the power is still an open question.

Nobody has yet argued a better model, but they should. Fine predictions about diffraction and interferences could be the touchstone.

10 Conclusion and experimental perspectives

The geometry of each Fermat's spindle-shaped channel is a spindle of revolution, generated by an arc of a circle secant around the optical axis, already defined in the 18th century, within the framework of geometric optics - obviously, several Fermat's spindle-shaped channels for a single quanton in cases of interference. The contradiction between these mathematical centers or Poncelet's points-limits for the wavefronts, and the diameters of the emission or absorption reactions is resolved by a slight offset of the mathematical apexes beyond the physical apexes.
The apparent contradiction in the diameters calculated over astronomical distances, which are visibly unrealistic, demonstrates that the individual character of the photons, however indisputable to the emission and absorption reactions, disappears in front of a bosonic collective of photons, sharing near frequencies and near wave vectors. The experimental truth test will be predicting in which area and with what light intensities the Hanbury Brown and Twiss grouping [9] will be evident, and then compare with the experimental data.

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